

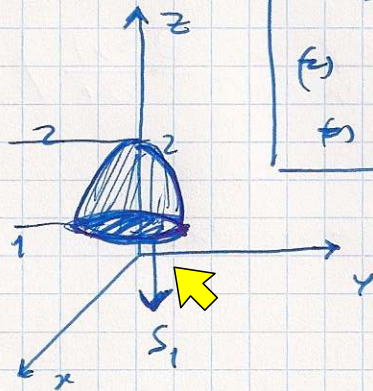
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$$F(x, y, z) = (x, y, z)$$

$$V: x^2 + y^2 \leq z \leq 2 - x^2 - y^2$$

$$\boxed{\begin{aligned} x^2 + y^2 &\leq z \\ \Leftrightarrow \rho^2 &\leq z \\ \Leftrightarrow \rho &\leq \sqrt{z} \end{aligned}}$$

$$\boxed{\begin{aligned} z &\leq 2 - x^2 - y^2 \\ \Leftrightarrow x^2 + y^2 &\leq 2 - z \\ \Leftrightarrow \rho &\leq \sqrt{2 - z} \end{aligned}}$$



$$\boxed{\theta \in [0, 2\pi]}$$

$$\boxed{1 \leq z \leq 2}$$

$$\sqrt{z} \leq \rho \leq \sqrt{2-z}$$

$$\left. \begin{aligned} x^2 + y^2 &= z \\ z &= 2 - x^2 - y^2 \end{aligned} \right\} \Rightarrow x^2 + y^2 = 1 \quad z = 1$$

$$\iint_S \vec{F} \cdot \vec{n}_{\text{ext}} dS + \iint_{S_1} \vec{F} \cdot \vec{n}_{\text{ext}} dS_1 = \iiint_V \nabla \cdot \vec{F} dx dy dz$$

$$\Leftrightarrow \iint_S \vec{F} \cdot \vec{n}_{\text{ext}} dS + \iint_{x^2+y^2 \leq 1} (x, y, z) \cdot (0, 0, -1) dx dy dz = \iiint_V 3 dx dy dz$$

$$\Leftrightarrow \iint_S \vec{F} \cdot \vec{n}_{\text{ext}} dS + \int_0^{2\pi} \int_0^1 \rho d\rho d\theta = \iiint_V 3 dx dy dz$$

$$\Leftrightarrow \iint_S \vec{F} \cdot \vec{n}_{\text{ext}} dS + \int_0^{2\pi} \left[\frac{\rho^2}{2} \right]_0^1 d\theta = \iiint_V 3 dx dy dz$$

$$\Leftrightarrow \iint_S \vec{F} \cdot \vec{n}_{\text{ext}} dS + \underbrace{\int_0^{2\pi} \frac{1}{2} d\theta}_{= \pi} = \underbrace{\iiint_V 3 dx dy dz}_{=?} \quad *$$

$$* \iiint_V 3 \, dx \, dy \, dz = 3 \int_0^{2\pi} \int_1^2 \int_{\sqrt{z}}^{\sqrt{2-z}} \rho \, d\rho \, dz \, d\theta$$

$$= 3 \int_0^{2\pi} \int_1^2 \left[\frac{\rho^2}{2} \right]_{\sqrt{z}}^{\sqrt{2-z}} dz \, d\theta$$

$$= 3 \int_0^{2\pi} \int_1^2 \left(\frac{2-z}{2} - \frac{z}{2} \right) dz \, d\theta$$

$$= 3 \int_0^{2\pi} \int_1^2 \left(\frac{2-2z}{2} \right) dz \, d\theta = 1 - z$$

$$= 3 \int_0^{2\pi} \left[z - \frac{z^2}{2} \right]_1^2 d\theta$$

$$= 3 \int_0^{2\pi} \left(\underbrace{2 - \frac{3}{2}}_0 - \left(1 - \frac{1}{2} \right) \right) d\theta$$

$$= 3 \int_0^{2\pi} -\frac{1}{2} d\theta$$

$$= -\frac{3}{2} \int_0^{2\pi} d\theta = -3\pi$$

$$\Rightarrow \iint_S \vec{F} \cdot \vec{n}_{out} \, dS = -4\pi$$